Crumple mediated snap-through and post-buckling behavior of cylindrically bent sheets

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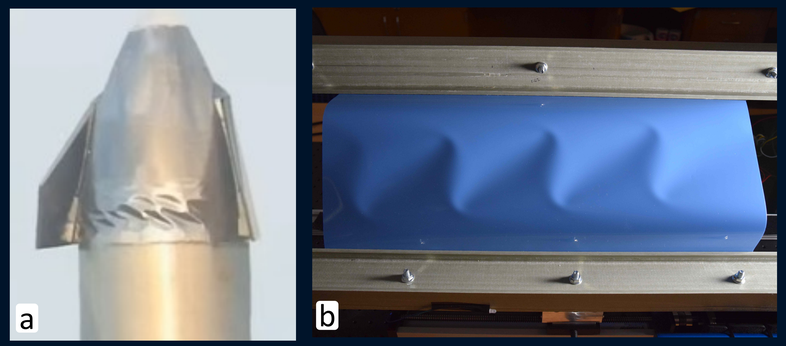
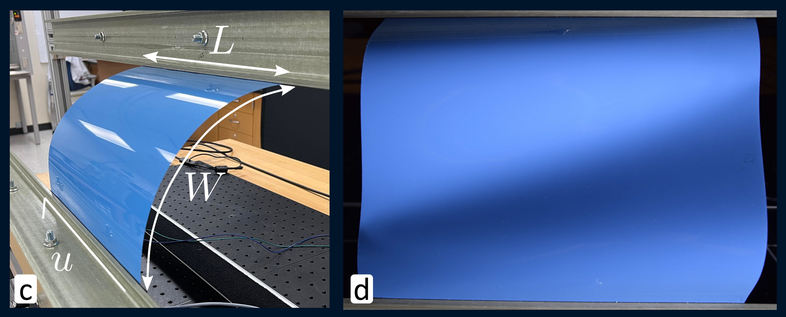
Abstract

Cylindrically bent sheets represent a common structural component in flight vessels. Asymmetric loading leading to different modes of buckling is common in these types of structures. While the plastic deformations which occur post-buckling have been well studied [1], we investigate the elastic, reversible buckling process which is mediated by localized regions of high elastic energy, or "crumples" [3].



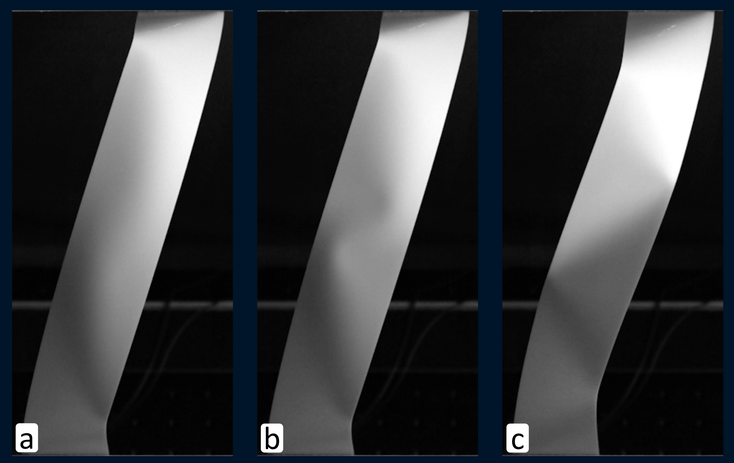
Comparison between our preliminary experimental results (b) and the buckling nose-cone of the SpaceX SN10 Starship [2] (a)



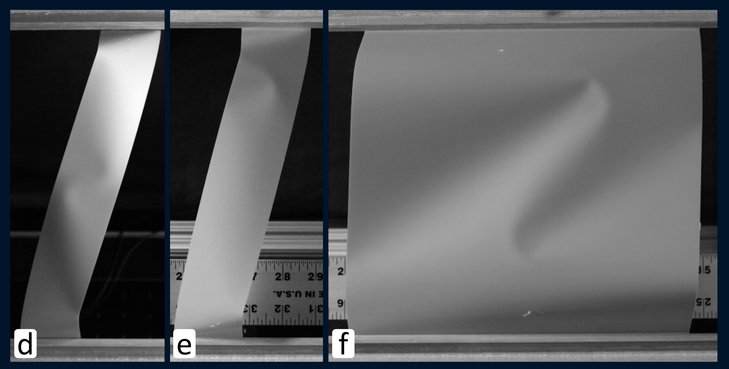
Cylindrically bent sheet (c) with length (L) and width (W) which is clamped on both ends and subjected to lateral end displacement (u), and the ultimate post-snap-through state where the sheet side profile forms an "S" rather than a "U" (d).

Transient crumples

In thin strips, characterized by aspect ratio $\alpha = L/W < 0.175$, no distinct bifurcation event is observed. When $0.175 \leq \alpha \leq 1.2$, the snap-through process is mediated by transient crumples.



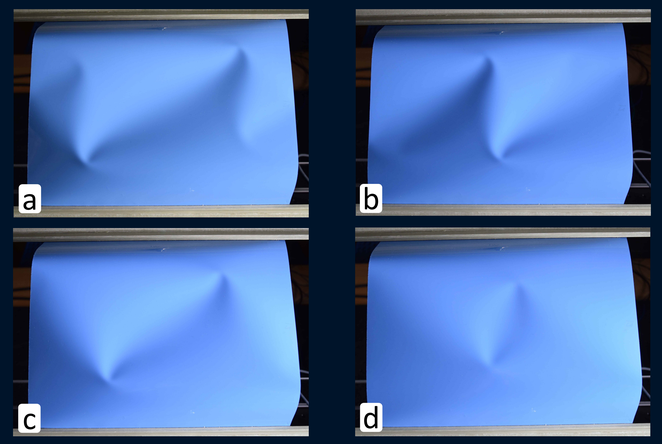
A sheet characterized by $\alpha = 0.175$ in the pre-buckling state (a), under transient crumpling (b), and post-snap-through state (c)



A collection of all observed transient crumples. In (d) $\alpha = 0.175$ and a small, center-originating pair of crumples mediates snap-through. In (e) $\alpha = 0.2$ and snap-through is instead mediated by a single edge-originating crumple. In (f), $\alpha = 1$ and snap-through is mediated by a pair of edge-originating crumples.

Stable states and critical u

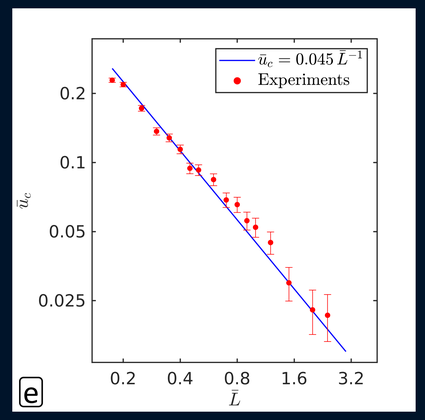
Above aspect ratio $\alpha = 1.2$, post-buckling stable states are observed. These states involve various arrangements of stable crumples depending on α.

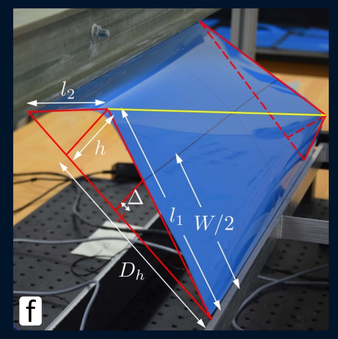


A collection of examples (a through d) where arrangements of crumples mediate stable states between reference configuration and complete snap-through

We employ a geometric argument that suggests that the critical displacement (first bifurcation) is:

$$u_c(\alpha) = \frac{2\sqrt{2}R\Delta}{\alpha}$$





Critical displacement as a function of α ($u_c(\alpha)$) (e), and the geometric argument which considers a compatibility condition in the distance h (f)

[1] Harris, L. A., Suer, H. S., & Skene, W. T. (1961). Model investigations of unstiffened and stiffened circular shells: Test techniques used in evaluation of the effect of internal pressure on stability of shells used in modern aircraft and missiles are described; also, conclusions derived from the results are presented and discussed. *Experimental Mechanics*, *1*, 1-9.

[2] Scott Manley. SpaceX's Starship SN10 Successfully Lands after Amazing Flight. Dismantles Itself Spectacularly, 2021. URL https://www.youtube.com/watch?v=CF9mdMI1qxM.

[3] Thomas A Witten. Stress focusing in elastic sheets. Reviews of Modern Physics, 2007.

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